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Application of a Finite-difference Solver with a Contraction Preconditioner to 3D EM Modeling in Mineral Exploration

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SUMMARY

This paper uses a novel approach to constructing an effective preconditioner for finite-difference (FD) electromagnetic modeling in applications to mineral exploration. This approach uses a FD contraction operator, similar to one developed for integral equation modeling. We demonstrate the effectiveness of this new modeling method and corresponding code in the sensitivity study of magnetotelluric data to an ore deposit, typical for Norilsk ore region.

Introduction

The choice of an efficient method of 3D numerical modeling is crucial for inversion of electromagnetic data. The most time-consuming step in finite-difference (FD) modeling is the solution of the corresponding system of linear equations. Both iterative and direct solvers have been used for EM modeling; however, direct solvers still impose challenging memory requirements for large-scale 3D problems, which make the iterative solvers more attractive. For the finite-difference discretization on nonuniform computational grids, we recently developed and analyzed a novel approach to constructing a preconditioned iterative solution of those systems. This approach relies on the energy equality, which states that all the energy emitted by the excess electric current inside the domain with the anomalous conductivity is converted into Joule heating within the computational domain.

We have applied this approach to the sensitivity study of magnetotelluric (MT) data with respect to an ore deposit. The tested resistivity model is based on a typical deposit of Norilsk ore region, which was studied recently by a detailed magnetotelluric survey supported by drilling.

Numerical Modeling Method

We consider a geoelectrical model with the spatial conductivity distribution, $\sigma(x,y,z)$, represented as a superposition of layered background conductivity, $\sigma_b(z)$, and anomalous conductivity, $\sigma_a(x,y,z)$, i.e. $\sigma(x,y,z) = \sigma_b(z) + \sigma_a(x,y,z)$. Given a particular source, a similar decomposition could be performed for electric fields as well,

$$\boldsymbol{E}(x,y,z) = \boldsymbol{E}_b(x,y,z) + \boldsymbol{E}_a(x,y,z),$$

where $\boldsymbol{E}(x,y,z)$, $\boldsymbol{E}_b(x,y,z)$, and $\boldsymbol{E}_a(x,y,z)$ are the total, background, and anomalous electric field respectively (Zhdanov, 2009). Since $\boldsymbol{E}_b(x,y,z)$ can be found quasi-analytically, the solution of the forward modeling problem can be reduced to solving for $\boldsymbol{E}_a(x,y,z)$. The well-known advantage of this aproach is that we can use the known solutions for the background field and solve the differential equations for the anomalous field only avoiding modelling a singularity in the source.

Following the conventional edge-based finite-difference discretization (Yee, 1966; Weiss and Newman, 2002), we introduce the discrete conductivities for the total, background and anomalous models, $\mathbf{\Sigma} = \mathbf{\Sigma}_b + \mathbf{\Sigma}_a$, as well as the respective discrete electric fields, $\mathbf{e} = \mathbf{e}_\mathrm{a} + \mathbf{e}_\mathrm{b}$. The linear system of algebraic equations corresponding to the differential equations for $\mathbf{e}_a$ has the following form:

$$\mathbf{A}\mathbf{e}_a = i\omega\mu_0\mathbf{\Sigma}_a\mathbf{e}_b, \tag{1}$$

where $\mathbf{A}$ is a square system matrix, ω is the source angular frequency, μ_0 is the magnetic permeability of the free space.

Let $\mathbf{A}_b$ be the FD system matrix, corresponding to the background conductivity model, $\sigma_b(z)$. Interestingly, this matrix can be implicitly factorized and the action of the inverse matrix can be efficiently computed. As a result, it can be used as a preconditioner to (1):

$$\mathbf{A}_b^{-1}\mathbf{A}\mathbf{e}_a = i\omega\mu_0\mathbf{A}_b^{-1}\mathbf{\Sigma}_a\mathbf{e}_b \tag{2}$$

The performance of this preconditioner depends on how close the total model is to the background model. We write the respective estimate as the following double inequality:

$$\alpha\sigma_b(z) \le \sigma(x,y,z) \le \beta\sigma_b(z), \quad 0 < \alpha \le 1 \le \beta.$$

This inequality ensures that the anomalous domains are neither perfect conductors nor insulators. It can be shown that the condition number of the matrix arising in (2) is essentially dictated by the following quotient,

$$\beta/\alpha. \tag{3}$$

This preconditioner will be referred to as the Green's function preconditioner (or FD 1D), since $\mathbf{A}_b^{-1}$ corresponds to the Green's function for the layered background model.

In order to minimize the dependence of the solver's complexity from the conductivity contrast, Pankratov et al. (1995), Singer (1995), suggested a transformation, which has the properties of a contraction integral equation operator (Zhdanov and Fang, 1997; Hursan and Zhdanov, 2002). For the FD formulation, it can be introduced as following. We define the modified FD Green's operator according to the following formula:

$$\mathcal{G}_b^M = 2i\omega\mu_0 \mathbf{\Sigma}_b^{\frac{1}{2}} \mathbf{A}_b^{-1} \mathbf{\Sigma}_b^{\frac{1}{2}} + \mathbf{I}. \tag{4}$$

Using this operator, equation (1) can be written in an equivalent form as follows:

$$\hat{\mathbf{e}}_a = \mathcal{G}_b^M \, \mathbf{K}_2 \mathbf{K}_1^{-1} \hat{\mathbf{e}}_a + i\omega\mu_0 \mathbf{\Sigma}_b^{\frac{1}{2}} \mathbf{A}_b^{-1} \mathbf{\Sigma}_a \mathbf{e}_b, \;\; \hat{\mathbf{e}}_a = \mathbf{K}_1 \mathbf{e}_a, \tag{5}$$

where $\mathbf{K}_1$, $\mathbf{K}_2$ are diagonal matrices,

$$\mathbf{K}_1 = \frac{1}{2}(\mathbf{\Sigma} + \mathbf{\Sigma}_b)\mathbf{\Sigma}_b^{-\frac{1}{2}}, \quad \mathbf{K}_2 = \frac{1}{2}(\mathbf{\Sigma} - \mathbf{\Sigma}_b)\mathbf{\Sigma}_b^{-\frac{1}{2}}. \tag{6}$$

By introducing a new operator,

$$\mathbf{C} = \mathcal{G}_b^M \, \mathbf{K}_2 \mathbf{K}_1^{-1}, \tag{7}$$

we rewrite system (5) as follows:

$$\hat{\mathbf{e}}_a = \mathbf{C}\hat{\mathbf{e}}_a + i\omega\mu_0 \mathbf{\Sigma}_b^{\frac{1}{2}} \mathbf{A}_b^{-1} \mathbf{\Sigma}_a \mathbf{e}_b. \tag{8}$$

Finally, we arrive at the following preconditioned system of equations for the scaled anomalous electric field, $\hat{\mathbf{e}}_a$:

$$(\mathbf{I} - \mathbf{C})\hat{\mathbf{e}}_a = i\omega\mu_0 \mathbf{\Sigma}_b^{\frac{1}{2}} \mathbf{A}_b^{-1} \mathbf{\Sigma}_a \mathbf{e}_b. \tag{9}$$

An important feature of the above equation is that operator $\mathbf{C}$ is a contraction operator for media of any contrast. A simple estimate for the condition number of $(\mathbf{I} - \mathbf{C})$ can be derived for high-contrast geoelectrical models, i.e. $0 < \alpha \ll 1$ and $\beta \gg 1$:

$$cond(\mathbf{I} - \mathbf{C}) \leq \max\left\{\frac{1}{\alpha}, \beta\right\}. \tag{10}$$

We will refer to this preconditioner as a contraction operator (CO) preconditioner. We can conclude from estimates (3) and (10) that, when both conductive and resistive anomalies are present in the geoelectrical model ($\alpha < 1$, $\beta > 1$), the solver based on the CO preconditioner will converge faster than the one based on the FD 1D preconditioner.

Sensitivity study of the MT responses

Figure 1 presents a resistivity model used in our sensitivity study. It is a schematized and horizontally scaled version of a detailed resistivity model of the ore-bearing intrusion of Norilsk ore region, constructed using the recently obtained MT data. The sedimentary background includes a near-surface layer and sedimentary host rocks. The resistive granite intrusion includes two conductive ore bodies, formed due to gravitational differentiation during magma cooling. We have estimated sensitivity of MT data to the presence of the ore bodies in the bottom of the intrusion.

This model features a dramatic resistivity contrast of hundred thousand times making modeling a challenging procedure since this contrast contributes to the condition number of the system matrix, $\mathbf{A}$, of the FD equations. For this model, iterative solution of equation (1) requires a robust preconditioner. We studied the performance of the two preconditioners introduced in the previous section.

We generated a 154×108×55 nonuniform computational grid with the smallest cell size of 15×15×15 m^3 for plane-wave excitation at a period of 10 s. Table 1 shows iteration count, N_{it}, and CPU time, t, in seconds of the BiCGStab (Saad, 2003) iterative solver needed to reach a residual relative norm of 1e-8, leveraged with the two preconditioners. Iteration count and CPU time are presented for both x- and y-polarisations.

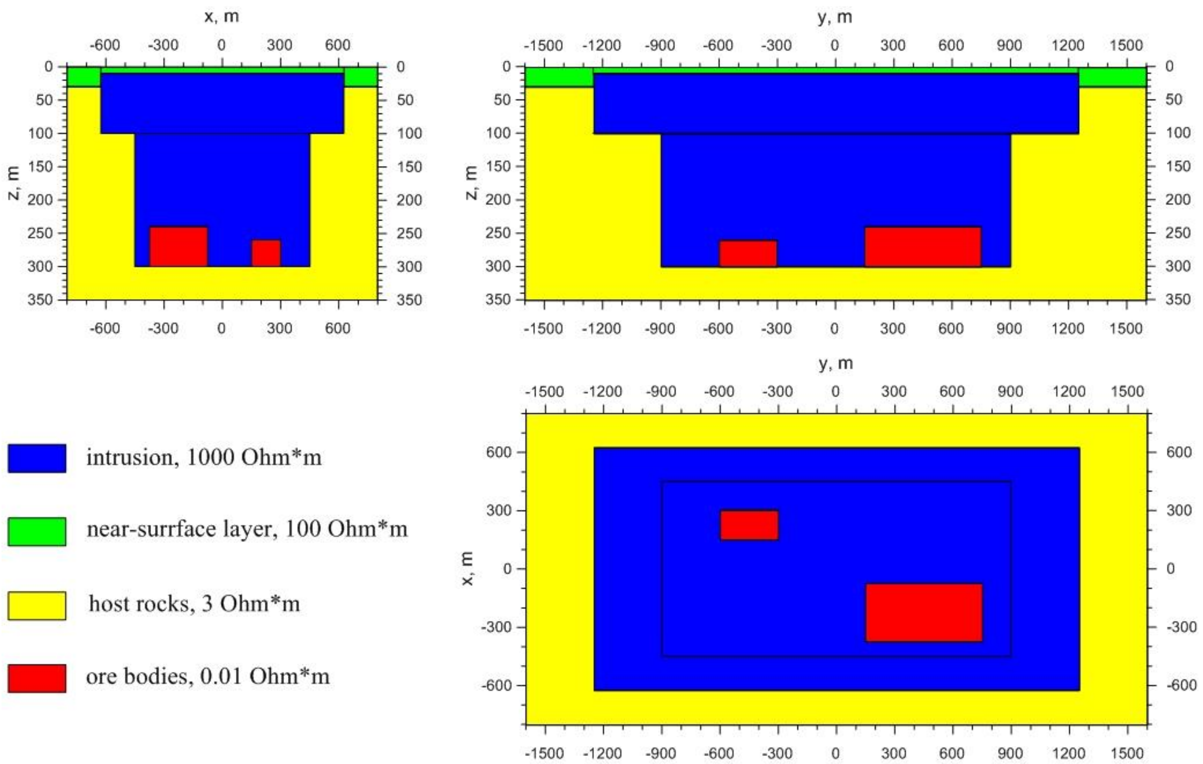

***Figure 1** Resistivity model of the ore-bearing intrusion projected onto x0z, y0z and x0y planes.*

We observed that the use of the CO preconditioner gave us a speedup of almost four times. This result is consistent with the analysis presented in the previous section. The model involves both resistive and conductive inclusions; therefore convergence of the iterative solver with the CO preconditioner was expected to be faster.

Finally, we present the pseudo-section plots of apparent resistivities, ρ_{xy} and ρ_{yx}, and impedance phases, ϕ_{xy} and ϕ_{yx}, along three profiles, $y = -450, 0, 450$ m (Figure 2). The data was computed at 21 periods, from 0.01 to 1000 s, four periods per decade. The separate grids were used for every period to make the smallest cell size at most half of the ore bodies' skin depth.

***Table 1** Iteration count, N_{it}, and CPU time, t, in seconds of the BiCGStab iterative solver.*

	x-polarisation		y-polarisation	
	FD 1D	CO	FD 1D	CO
N_{it}	922	230	871	220
t, s	4306	1097	4067	1056

Figure 2 indicates a pronounced impact of the larger ore body on the responses. On the other hand, a delineation of the smaller body may require high-quality data and/or inversion. The larger body has six times larger volume, it is also 50 % thicker, than the smaller one; therefore its better sensitivity should be expected.

Conclusions

In this paper, we have studied performance of the two preconditioners applied to the finite-difference system of equations used in 3D MT modeling. We have demonstrated both analytically and numerically that the iterative solver preconditioned with the contraction operator converges faster than the one with Green's function preconditioner on some models. Specifically, a speedup was proved to take place for high-contrast models involving both resistive and conductive inclusions. We illustrated our discussion with sensitivity study of the MT data to ore bodies included into a granite intrusion.

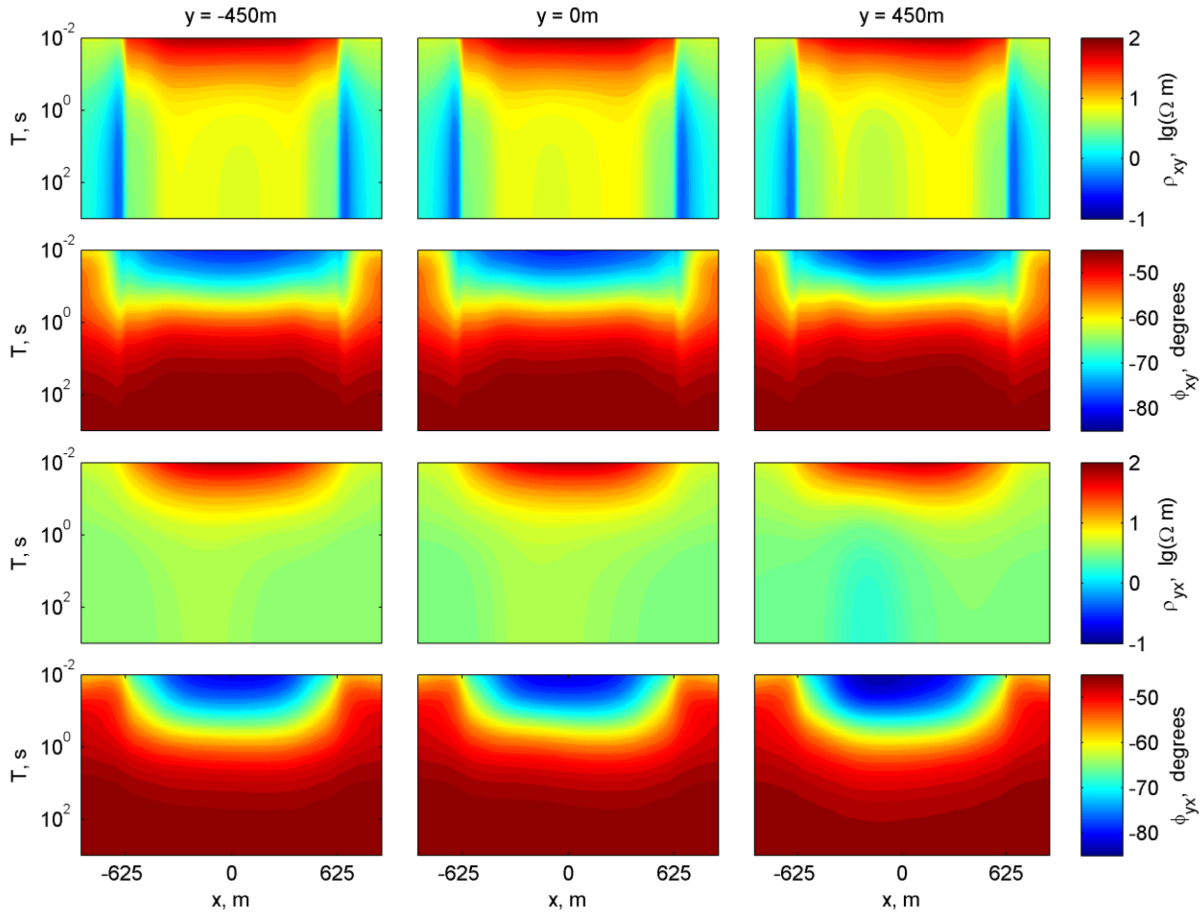

Figure 2 *Apparent resistivities(common logarithm) and impedance phases along three profiles.*

Acknowledgments

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